A linear operator framework for analysing resource usage

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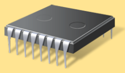
Aim : modelling and estimating resource usage of software

Generic model for resources


energy


memory


time

Quantitative semantics

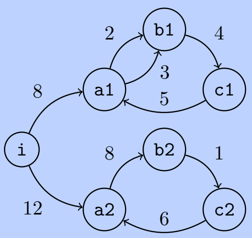
$q(i, a_2, b_2) = 12 \otimes 8$
Accumulation of costs along a path

$q(a_1, b_1) = 2 \oplus 3$
Maximum cost between two paths

$\sqrt[3]{q(i, a_1, b_1, c1)} = \sqrt[3]{8 \otimes (2 \oplus 3) \otimes 4}$
Average cost of a path

$\mathbb{1}$: neutral cost

$\mathbb{0}$: impossibility of a transition



Labelled transition system

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

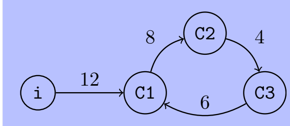
Transition system as a matrix M :
linear operator semantics

Hint : see $\oplus$ and $\otimes$
as max and + over reals

$$\rho(M) = \bigoplus_{k=1}^{|\Sigma|} \sqrt[k]{tr\, M^k}$$

Long-run cost
maximum average along traces

Abstraction



Abstract transition system

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Abstraction matrix α

α

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 6 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

Abstract semantics $M^\sharp$

The concrete semantics matrix M is abstracted into a smaller one $M^\sharp$ thanks to an abstraction operator α that merges states

Correctness condition for an abstraction

$$\alpha \circ M \leq M^\sharp \circ \alpha$$

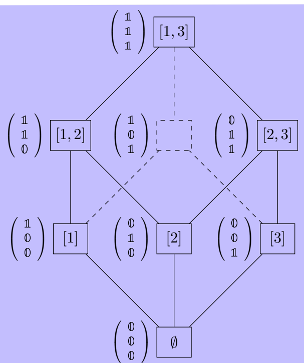
Derivation of the « best » abstract semantics :

$$M^\sharp = \alpha \circ M \circ \alpha^{-1}$$

Soundness of abstractions

Theorem : $\rho(M) \leq \rho(M^\sharp)$

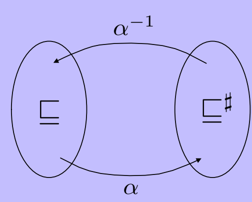
Correct abstractions safely over-approximate long-run cost computations



Example of a set lattice and its associated powerset

Improvements in abstraction design

- reconciliating classical and quantitative linear models
- reusing well-known abstractions (intervals, polyhedra ...)
- vector representation that takes two order relations into account



Classical abstract interpretation

- concrete and abstract semantics are linked via a Galois connection
- partial order on states

Application : computing in various energy modes

$$\frac{pc \ \texttt{setmode}\ m' \ pc'}{(pc, \rho, m) \rightarrow^{c_m(m,m')} (pc', \rho, m')}$$

Quantitative semantics rule

```
setsize b n;
setsize tmp n;
if n < 100 then setmode A else setmode D
   apply copy_to b, tmp;
```

Explicit energy modes management

Energy mode	Time (in cycles)	Cost (wrt array size)
A	5	0.2
C	2	$\lambda n.n$
D	1	$\lambda n.3.*n$
E	0	$\lambda n.5.*n$

Cost of an array operation

Energy mode		Threshold (array size)		
High	Low	100	500	900
A	B	3.6	3.6	3.6
A	C	13.7	50.9	88.1
A	D	39.9	191.6	343.3
A	E	81.2	405.1	728.9
B	C	13.7	50.9	88.1
B	D	39.9	191.6	343.3
B	E	81.2	405.1	728.9
C	D	97.5	191.6	343.3
C	E	97.5	405.1	728.9
D	E	381.6	405.1	728.9

Example of experimental results