Stochastic Process Algebras and Ordinary Differential Equations

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University of Edinburgh

5th September 2011

Outline

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Process Algebra

- Models consist of agents which engage in actions.

$$\alpha.P$$

action type or name → α

agent/ component → P

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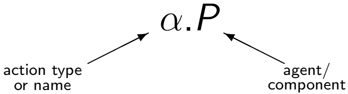

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Process algebra model $\xrightarrow{\text{SOS rules}}$ Labelled transition system

Stochastic process algebras

Process algebras where models are decorated with quantitative information used to generate a stochastic process are stochastic process algebras (SPA).

Stochastic Process Algebra

- Models are constructed from components which engage in activities.

$$(\alpha, r).P$$

action type
or name

activity rate
(parameter of an
exponential distribution)

component/
derivative

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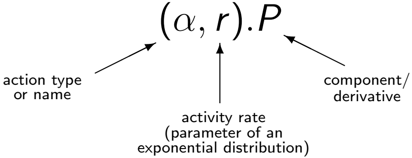

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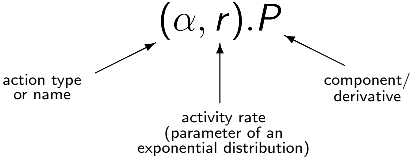

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- Moreover properties of that mathematical structure may be deduced by the construction at the process algebra level.
- Furthermore formal reasoning techniques such as equivalence relations and model checking can be used to manipulate or interrogate models.
- Compositionality can be exploited both for model construction and (in some cases) for model analysis.

Performance Evaluation Process Algebra (PEPA)

$(\alpha, f).P$	Prefix
$P_1 + P_2$	Choice
$P_1 \underset{L}{\bowtie} P_2$	Co-operation
P/L	Hiding
X	Variable

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When working with large numbers of entities, we write $P[n]$ to denote an array of n copies of P executing in parallel.

$$P[5] \equiv (P \parallel P \parallel P \parallel P \parallel P)$$

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Stochastic process algebras differ in how they define the rate of synchronised actions. In PEPA cooperation between components gives rise to shared actions, the rate of which are governed by the assumption of bounded capacity.

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Thus shared actions proceed at the minimum of the rates in the participating components.

In contrast independent actions do not constrain each other and if there are multiple copies of a action enabled in independent concurrent components their rates are summed.

A simple example: processors and resources

$$
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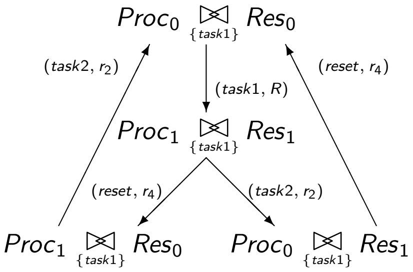

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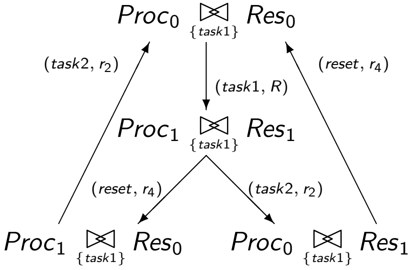

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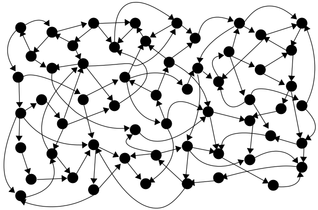
$$
\mathbf{Q} = \begin{pmatrix}
-R & R & 0 & 0 \\
0 & -(r_2 + r_4) & r_4 & r_2 \\
r_2 & 0 & -r_2 & 0 \\
r_4 & 0 & 0 & -r_4
\end{pmatrix}
$$

Solving discrete state models

As we have seen the SOS semantics of a SPA model is mapped to a CTMC with global states determined by the local states of the participating components.



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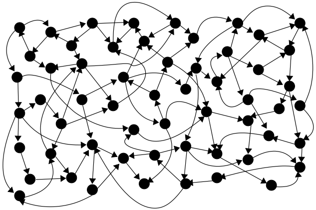

$$Q = \begin{pmatrix} q_{1,1} & q_{1,2} & \cdots & q_{1,N} \\ q_{2,1} & q_{2,2} & \cdots & q_{2,N} \\ \vdots & \vdots & & \vdots \\ q_{N,1} & q_{N,2} & \cdots & q_{N,N} \end{pmatrix}$$

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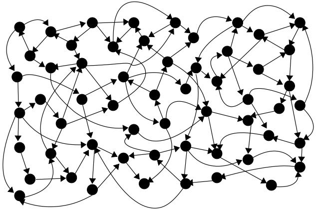
As we have seen the SOS semantics of a SPA model is mapped to a CTMC with global states determined by the local states of the participating components.

When the size of the state space is not too large they are amenable to numerical solution (linear algebra) to determine a steady state or transient probability distribution.



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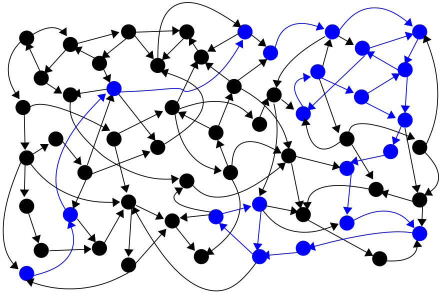
$$\pi(t) = (\pi_1(t), \pi_2(t), \ldots, \pi_N(t))$$

Solving discrete state models

Alternatively they may be studied using stochastic simulation. Each run generates a single trajectory through the state space. Many runs are needed in order to obtain average behaviours.



Benefits of process algebra

Beyond the clear benefits for model construction to be derived from using a high-level language with compositionality the formal nature of the process algebra specification has been exploited in a number of ways.

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For example,

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- The correspondence between the congruence, Markovian bisimulation, in the process algebra and the lumpability condition in the CTMC, allows exact model reduction to be carried out compositionally.
- Characterisation of product form structure at the process algebra level allows decomposed model solution based on the process algebra structure of the model.
- Stochastic model checking based on the CSL family of temporal logics allows evaluation of quantified properties of the behaviour of the system

Disadvantages of process algebra

The primary disadvantage of stochastic process algebras, shared by all discrete event modelling paradigms, is the problem of state space explosion, also known as the curse of dimensionality.

Simple example revisited

$$\begin{aligned} Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\ Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\ Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\ Res_1 &\stackrel{def}{=} (reset, r_4).Res_0 \end{aligned}$$

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CTMC interpretation

Processors (N_P)	Resources (N_R)	States ($2^{N_P+N_R}$)
1	1	4
2	1	8
2	2	16
3	2	32
3	3	64
4	3	128
4	4	256
5	4	512
5	5	1024
6	5	2048
6	6	4096
7	6	8192
7	7	16384
8	7	32768
8	8	65536
9	8	131072
9	9	262144
10	9	524288
10	10	1048576

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Large scale software systems
Issues of scalability are important for user satisfaction and resource efficiency but such issues are difficult to investigate using discrete state models.

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Biochemical signalling pathways
Understanding these pathways has the potential to improve the quality of life through enhanced drug treatment and better drug design.

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Epidemiological systems
Improved modelling of these systems could lead to improved disease prevention and treatment in nature and better security in computer systems.

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Crowd dynamics
Technology enhancement is creating new possibilities for directing crowd movements in buildings and urban spaces, for example for emergency egress, which are not yet well-understood.

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But solution techniques which rely on explicitly building the state space, such as numerical solution, are hampered by space complexity...

...whilst those that use the implicit state space, such as simulation, run into problems of time complexity.

The CODA project

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However, our main approach has been to use a counting abstraction in order to make a shift to population statistics and to develop a fluid approximation.

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To characterise the behaviour of a population we count the number of individuals within the population that are exhibiting certain behaviours rather than tracking individuals directly.

Then we make a continuous approximation of how the counts vary over time.

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Linking process algebra and continuous mathematical models for dynamic evaluation represents a paradigm shift in how such formal models are analysed.

The prospect of formally-based quantified evaluation of dynamic behaviour could have significant impact in application domains which have traditionally worked directly at the level of fitting differential equation models.

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Alternative Representations

Large PEPA model → explicit state CTMC

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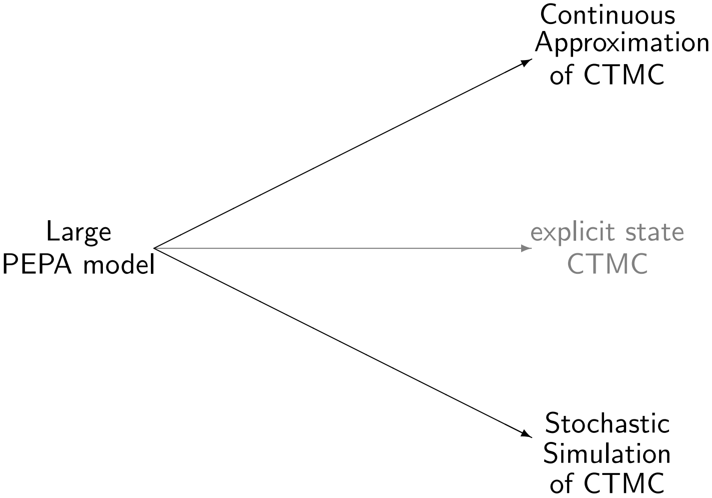

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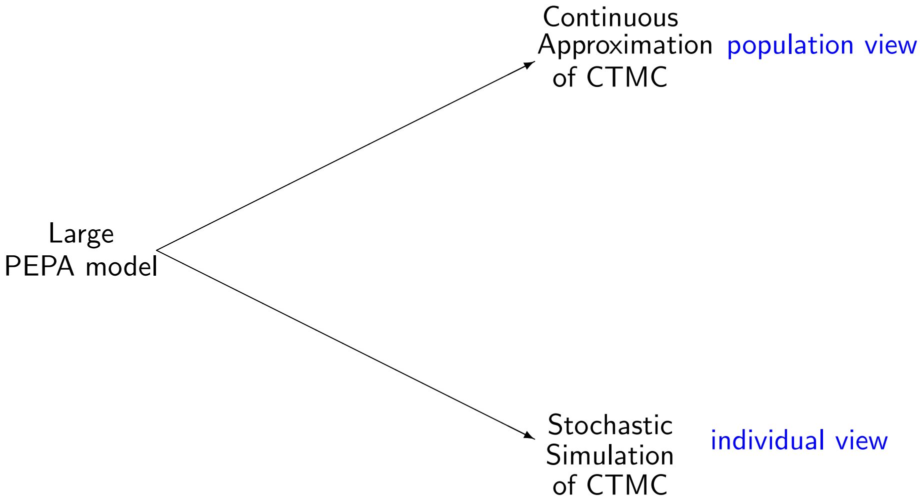

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1. Use a counting abstraction rather than the CTMC complete state space.
2. Assume that these state variables are subject to continuous rather than discrete change.
3. No longer aim to calculate the probability distribution over the entire state space of the model.
4. Instead the trajectory of the ODEs estimates the expected behaviour of the CTMC.

Models suitable for counting abstraction

- In the PEPA language multiple instances of components are represented explicitly — we write $P[n]$ to denote an array of n copies of P executing in parallel.

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- The impact of an action of a counting variable is
  - decrease by 1 if the component participates in the action
  - increase by 1 if the component is the result of the action
  - zero if the component is not involved in the action.

Simple example revisited

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- *task*1 decreases $Proc_0$ and Res_0
- *task*1 increases $Proc_1$ and Res_1
- *task*2 decreases $Proc_1$
- *task*2 increases $Proc_0$
- *reset* decreases Res_1
- *reset* increases Res_0

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$$Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]$$

$$\frac{dx_1}{dt} = -\min(r_1\, x_1, r_3\, x_3) + r_2\, x_2$$

$$x_1 = \text{no. of } Proc_1$$

- $task1$ decreases $Proc_0$
- $task1$ is performed by $Proc_0$ and Res_0
- $task2$ increases $Proc_0$
- $task2$ is performed by $Proc_1$

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ODE interpretation

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& \quad x_1 = \text{no. of } Proc_1 \\
\frac{dx_2}{dt} &= \min(r_1\, x_1, r_3\, x_3) - r_2\, x_2 \\
& \quad x_2 = \text{no. of } Proc_2 \\
\frac{dx_3}{dt} &= -\min(r_1\, x_1, r_3\, x_3) + r_4\, x_4 \\
& \quad x_3 = \text{no. of } Res_0 \\
\frac{dx_4}{dt} &= \min(r_1\, x_1, r_3\, x_3) - r_4\, x_4 \\
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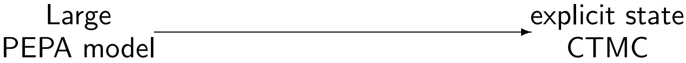

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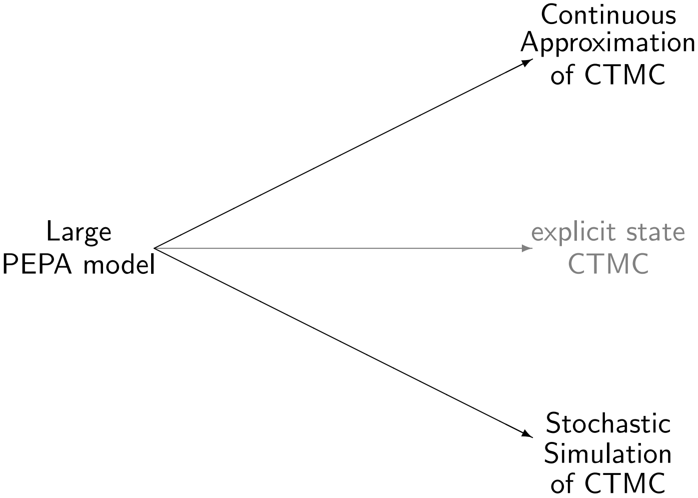

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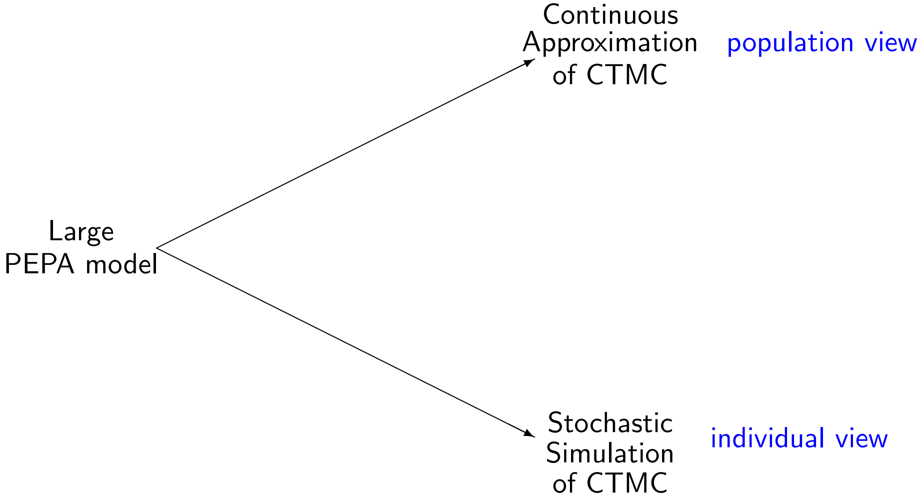

Alternative Representations

set of ODEs

Continuous Approximation of CTMC

population view

full generator matrix

Stochastic Simulation of CTMC

individual view

Alternative Representations

set of ODEs	Continuous Approximation of CTMC	population view
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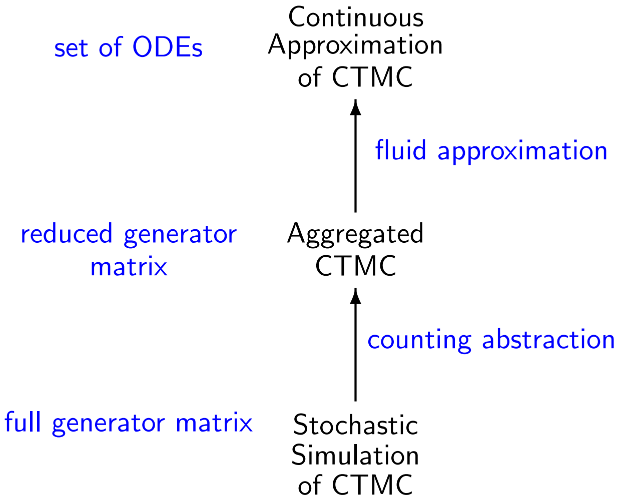
Alternative Representations



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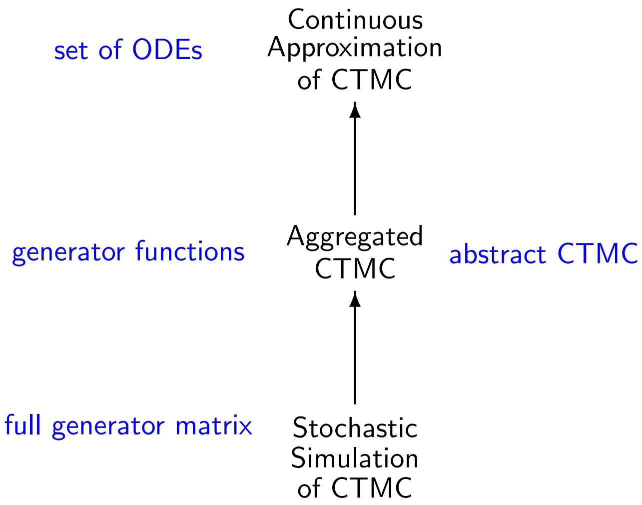

Outline

Deriving a Fluid Approximation of a PEPA model

The aim is to represent the CTMC implicitly (avoiding state space explosion), and to generate the set of ODEs which are the fluid limit of that CTMC.

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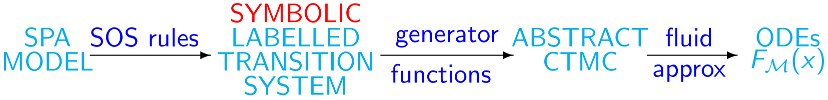

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In order to get to the implicit representation of the CTMC we need to:

1. Make the counting abstraction (Context Reduction)
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3. Calculate the rate of the transitions in terms of an arbitrary state of the CTMC.

Once this is done we can extract the vector field $F_{\mathcal{M}}(x)$ from the jump multiset.

Context Reduction

$$\begin{aligned}
Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
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System &\stackrel{def}{=} Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R]
\end{aligned}$$

$$\Downarrow$$

$$\mathcal{R}(System) = \{Proc_0, Proc_1\} \underset{\{task1\}}{\bowtie} \{Res_0, Res_1\}$$

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Population Vector

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

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Population Vector

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)$$

Fluid Structured Operational Semantics by Example

$$\begin{aligned}
\mathit{Proc_0} &\stackrel{\mathit{def}}{=} (\mathit{task1}, r_1).\mathit{Proc_1} \\
\mathit{Proc_1} &\stackrel{\mathit{def}}{=} (\mathit{task2}, r_2).\mathit{Proc_0} \\
\mathit{Res_0} &\stackrel{\mathit{def}}{=} (\mathit{task1}, r_3).\mathit{Res_1} \\
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$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

$$\frac{\dfrac{Proc_0 \xrightarrow{task1, r_1} Proc_1}{Proc_0 \xrightarrow{task1, r_1 \xi_1}_* Proc_1}}{}$$

Fluid Structured Operational Semantics by Example

$$\begin{aligned} Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\ Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\ Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\ Res_1 &\stackrel{def}{=} (reset, r_4).Res_0 \\ System &\stackrel{def}{=} Proc_0[N_P] \underset{\{task1\}}{\bowtie} Res_0[N_R] \end{aligned}$$

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\end{aligned}
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$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$$

$$
\frac{\dfrac{Proc_0 \xrightarrow{task1, r_1} Proc_1}{Proc_0 \xrightarrow{task1, r_1 \xi_1}_* Proc_1} \qquad \dfrac{Res_0 \xrightarrow{task1, r_3} Res_1}{Res_0 \xrightarrow{task1, r_3 \xi_3}_* Res_1}}{Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1}
$$

Apparent Rate Calculation

$$\dfrac{\dfrac{\mathit{Proc_0} \xrightarrow{\mathit{task1}, r_1} \mathit{Proc_1}}{\mathit{Proc_0} \xrightarrow{\mathit{task1}, r_1 \xi_1}_* \mathit{Proc_1}} \qquad \dfrac{\mathit{Res_0} \xrightarrow{\mathit{task1}, r_3} \mathit{Res_1}}{\mathit{Res_0} \xrightarrow{\mathit{task1}, r_3 \xi_3}_* \mathit{Res_1}}}{\mathit{Proc_0} \underset{\{\mathit{task1}\}}{\bowtie} \mathit{Res_0} \xrightarrow{\mathit{task1}, r(\xi)}_* \mathit{Proc_1} \underset{\{\mathit{task1}\}}{\bowtie} \mathit{Res_1}}$$

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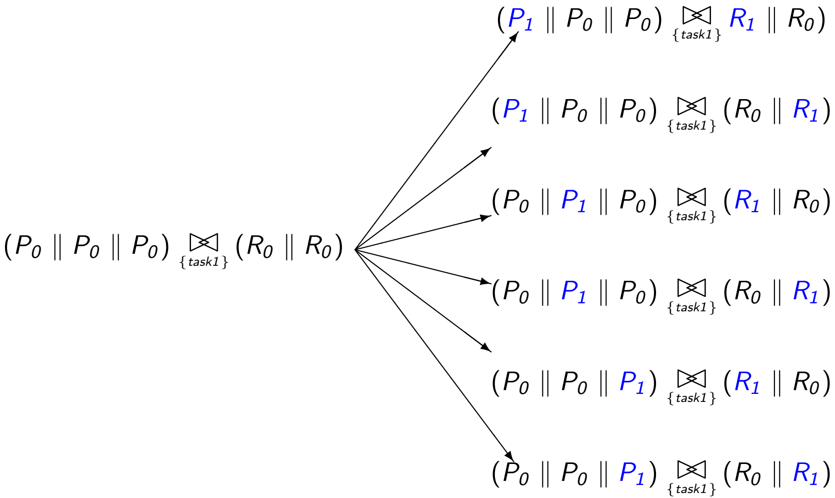
$$r(\xi) = \min\left(r_1 \xi_1, r_3 \xi_4\right)$$

$f(\xi, l, \alpha)$ as the Generator Matrix of the *Lumped* CTMC



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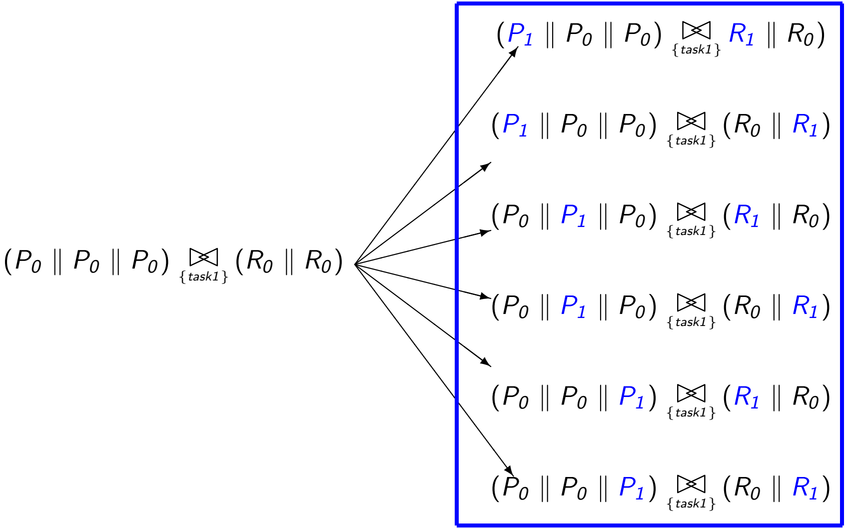

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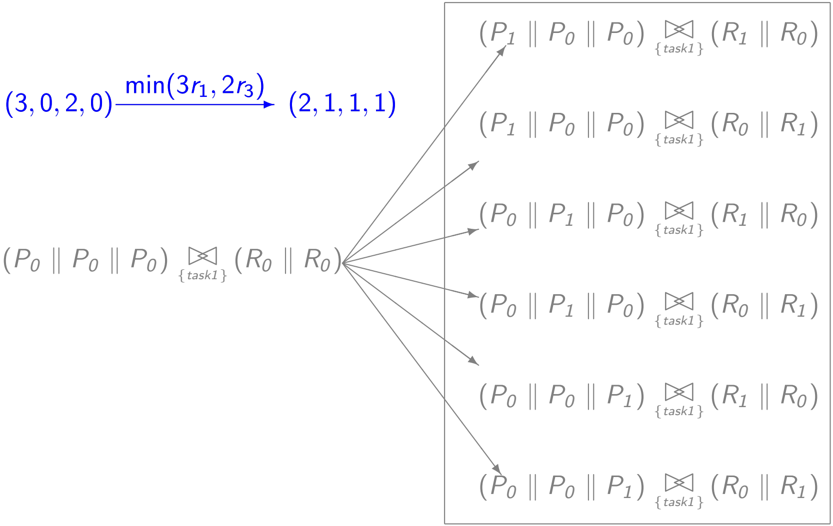

Jump Multiset

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$

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$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_{*} Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

Construction of $f(\xi, l, \alpha)$

$$\mathit{Proc_0} \underset{\{\mathit{task1}\}}{\bowtie} \color{blue}{\mathit{Res_1}} \xrightarrow{\mathit{reset}, \xi_4 r_4}_* \mathit{Proc_0} \underset{\{\mathit{task1}\}}{\bowtie} \color{blue}{\mathit{Res_0}}$$

Construction of $f(\xi, I, \alpha)$

$$\mathit{Proc_0} \underset{\{\mathit{task1}\}}{\bowtie} \mathit{Res_1} \xrightarrow{\mathit{reset}, \xi_4 r_4}_* \mathit{Proc_0} \underset{\{\mathit{task1}\}}{\bowtie} \mathit{Res_0}$$

- Take $I = (0, 0, 0, 0)$

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$$Proc_0 \underset{\{task1\}}{\bowtie} Res_1 \xrightarrow{reset, \xi_4 r_4}_* Proc_0 \underset{\{task1\}}{\bowtie} Res_0$$

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$$I = (-1 + 1, 0, +1, -1) = (0, 0, +1, -1)$$

Construction of $f(\xi, I, \alpha)$

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Construction of $f(\xi, l, \alpha)$

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Construction of $f(\xi, l, \alpha)$

$$\begin{array}{ccc} Proc_0 \underset{\{task1\}}{\bowtie} Res_0 & \xrightarrow{task1, r(\xi)}_* & Proc_1 \underset{\{task1\}}{\bowtie} Res_1 \\ Proc_1 \underset{\{task1\}}{\bowtie} Res_0 & \xrightarrow{task2, \xi_2 r_2'}_* & Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \end{array}$$

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Construction of $f(\xi, l, \alpha)$

$$Proc_0 \underset{\{task1\}}{\bowtie} Res_0 \xrightarrow{task1, r(\xi)}_* Proc_1 \underset{\{task1\}}{\bowtie} Res_1$$
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Capturing behaviour in the Generator Function

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Numerical Vector Form

$$\xi = (\xi_1, \xi_2, \xi_3, \xi_4) \in \mathbb{N}^4, \quad \xi_1 + \xi_2 = N_P \text{ and } \xi_3 + \xi_4 = N_R$$

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Generator Function

$$
f(\xi, l, \alpha): \quad
\begin{aligned}
f(\xi, (-1, 1, -1, 1), task1) &= \min(r_1\xi_1, r_3\xi_3) \\
f(\xi, (1, -1, 0, 0), task2) &= r_2\xi_2 \\
f(\xi, (0, 0, 1, -1), reset) &= r_4\xi_4
\end{aligned}
$$

Extraction of the ODE from f

Generator Function

$$f(\xi, l, \alpha): \quad \begin{array}{rcl} f(\xi,(-1,1,-1,1),\mathit{task1}) &=& \min\left(r_1\xi_1, r_3\xi_3\right) \\ f(\xi,(1,-1,0,0),\mathit{task2}) &=& r_2\xi_2 \\ f(\xi,(0,0,1,-1),\mathit{reset}) &=& r_4\xi_4 \end{array}$$

Differential Equation

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= F_{\mathcal{M}}(x) = \sum_{l \in \mathbb{Z}^d} l \sum_{\alpha \in \mathcal{A}} f(x, l, \alpha) \\ &= (-1,1,-1,1)\min\left(r_1x_1, r_3x_3\right) + (1,-1,0,0)r_2x_2 \\ &\qquad + (0,0,1,-1)r_4x_4 \end{aligned}$$

Extraction of the ODE from f

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Differential Equation

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = -\min\left(r_1x_1, r_3x_3\right) + r_2x_2$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = \min\left(r_1x_1, r_3x_3\right) - r_2x_2$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}t} = -\min\left(r_1x_1, r_3x_3\right) + r_4x_4$$

$$\frac{\mathrm{d}x_4}{\mathrm{d}t} = \min\left(r_1x_1, r_3x_3\right) - r_4x_4$$

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- We can prove this using Kurtz's theorem:
Solutions of Ordinary Differential Equations as Limits of Pure Jump Markov Processes, T.G. Kurtz, J. Appl. Prob. (1970).
- Moreover Lipschitz continuity of the vector field guarantees existence and uniqueness of the solution to the initial value problem.

Outline

Why use a process algebra?

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- Furthermore formal reasoning techniques such as equivalence relations and model checking can be used to manipulate or interrogate models.
- Compositionality can be exploited both for model construction and (in some cases) for model analysis.

Conclusions

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Conclusions

- Many interesting and important systems can be studied as a result of the link between stochastic process algebras and ODEs.
- Particularly, we can now consider examples of collective dynamics and emergent behaviour.
- Stochastic process algebras, are well-suited to modelling the behaviour of such systems in terms of the individuals and their interactions.
- Continuous approximation allows a rigorous mathematical analysis of the average behaviour of such systems.

On-going work

- Time series plots counting the populations of components over time tell us a great deal about the dynamics of the system but are not necessarily the information we require.

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- Future work will also consider exploiting more of the formal structure of the process algebras to assist in the manipulation and analysis of the ODEs.

Thanks!

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Acknowledgements: collaborators
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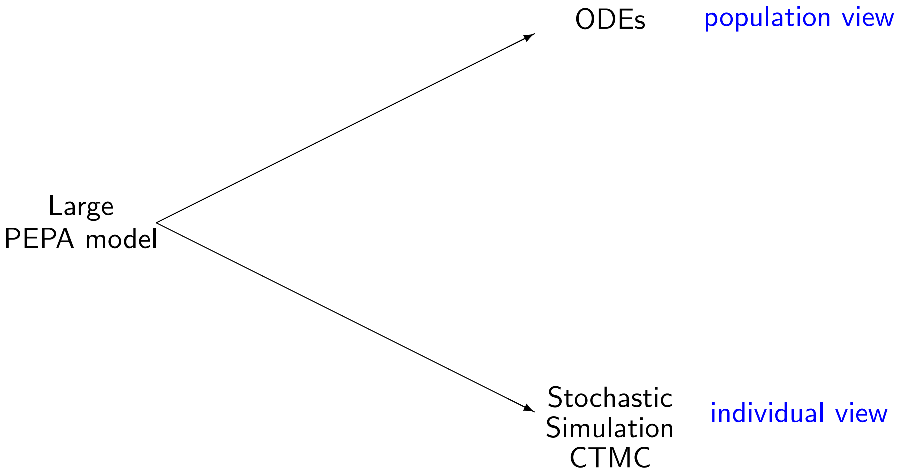
More information:

http://www.dcs.ed.ac.uk/pepa

Alternative Representations



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